

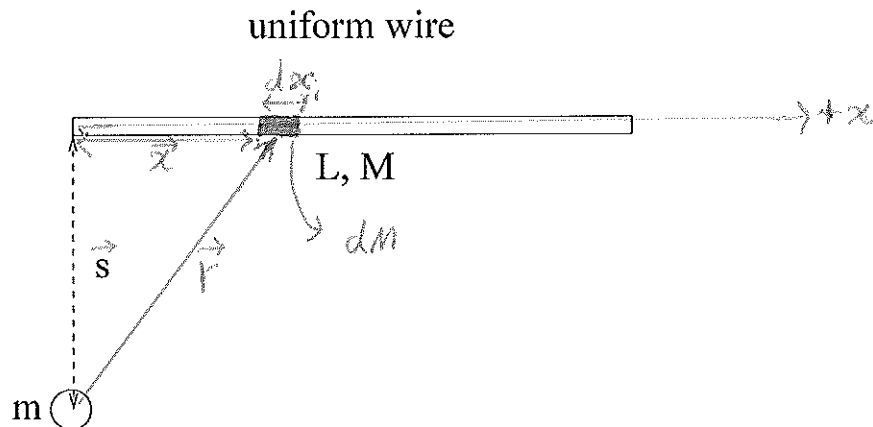
Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name:

Student ID:

Signature:

Find the potential energy between the uniform rod (length L and mass M) and the point mass m . (You don't have to evaluate the resultant integrals.)



dV : potential energy between and dM

$$\begin{array}{c|c} \text{length} & \text{mass} \\ \hline L & M \\ \hline dx & dM \end{array} \Rightarrow dm = \frac{M}{L} dx$$

$$dV = -G \frac{m dM}{r} = -G \frac{m \left(\frac{M}{L}\right) dx}{\sqrt{s^2 + x^2}}$$

$$\Rightarrow V = \int dV = \int_0^L \frac{-G m M dx}{L \sqrt{s^2 + x^2}} = -\frac{G m M}{L} \int_0^L \frac{dx}{\sqrt{s^2 + x^2}}$$

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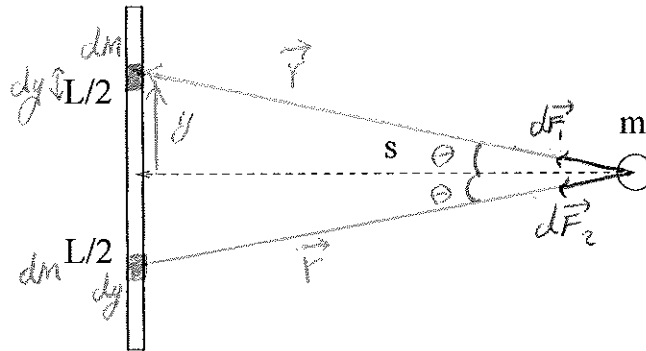
Name:

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Find the force between the uniform rod (length L and mass M) and the point mass m .
(You don't have to evaluate the resultant integrals.)

uniform rod



Due to symmetry $d\vec{F}_1 = d\vec{F}_2$

$$d\vec{F} = d\vec{F}_1 + d\vec{F}_2 \Rightarrow dF = 2dF_1 \cos\theta, \quad dm = \frac{M}{L} dy$$

$$dF_1 = G \frac{m dm}{r^2} = G \frac{m \cdot \frac{M}{L} dy}{y^2 + s^2} = \frac{GmM}{L} \frac{dy}{s^2 + y^2}$$

$$\Rightarrow dF = 2dF_1 \cos\theta = \frac{2GmM}{L} \frac{\cos\theta dy}{s^2 + y^2}, \quad \cos\theta = \frac{s}{r} = \frac{s}{\sqrt{s^2 + y^2}}$$

$$\Rightarrow dF = \frac{2GmM}{L} \frac{s dy}{(s^2 + y^2)^{3/2}}$$

$$\Rightarrow F = \int dF = \frac{2GmMs}{L} \int_{-L/2}^{L/2} \frac{dy}{(s^2 + y^2)^{3/2}}$$

To students why not
 $\int_{-L/2}^{L/2}$?
-L/2

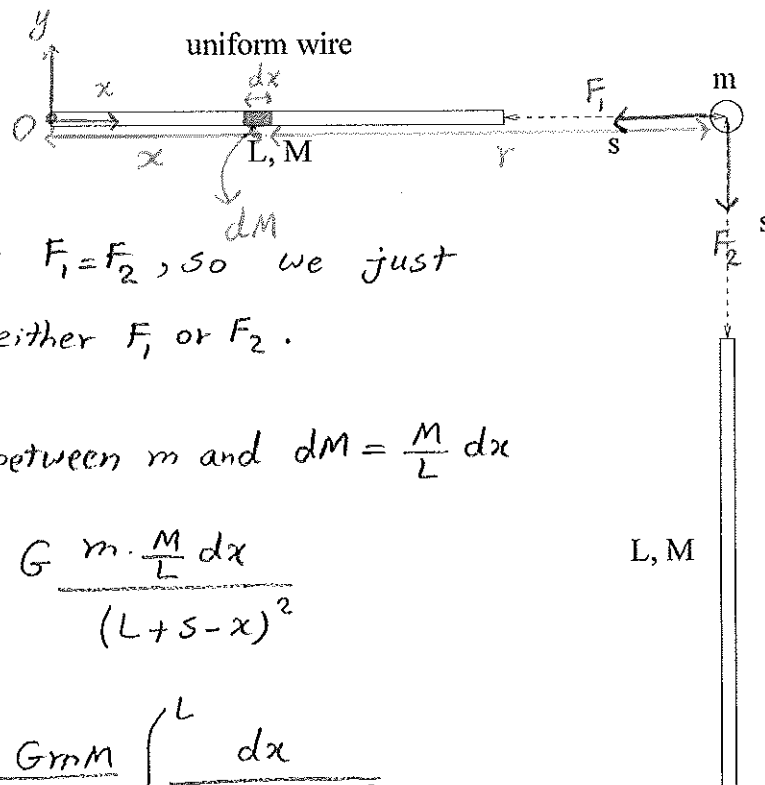
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Find the force between the uniform rod (length L and mass M) and the point mass m .
(You don't have to evaluate the resultant integrals.)



• Due to symmetry $F_1 = F_2$, so we just need to obtain either F_1 or F_2 .

dF_1 : the force between m and $dM = \frac{M}{L} dx$

$$dF_1 = G \frac{m dM}{r^2} = G \frac{m \cdot \frac{M}{L} dx}{(L+s-x)^2}$$

$$\Rightarrow F_1 = \int dF_1 = \frac{G m M}{L} \int_0^L \frac{dx}{(L+s-x)^2}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$|\vec{F}| = 2 F_1 \cos\left(\frac{90}{2}\right) = 2 F_1 \cos 45 = 2 F_1 \times \frac{\sqrt{2}}{2} = \sqrt{2} F_1$$

$$\Rightarrow |\vec{F}| = \frac{\sqrt{2} G m M}{L} \int_0^L \frac{dx}{(L+s-x)^2}$$

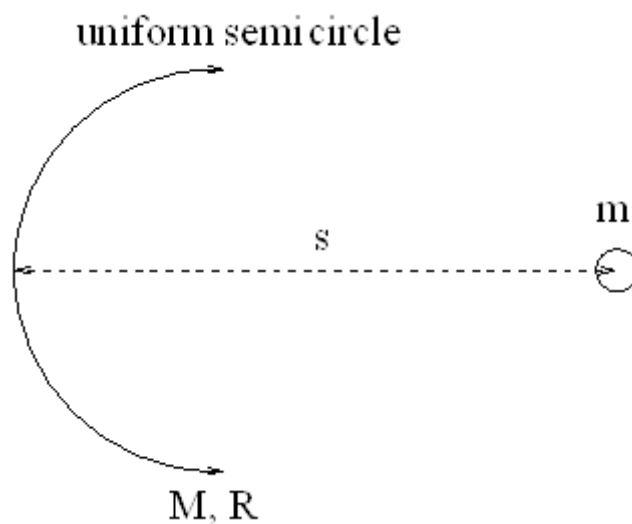
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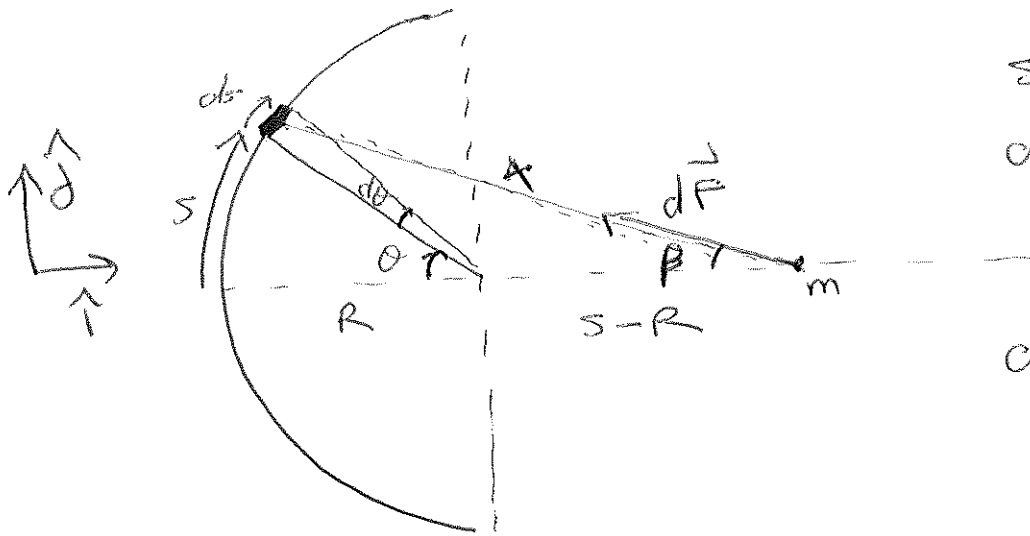
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Find the force between the uniform semi-circular wire (radius R and mass M) and the point mass m . (You don't have to evaluate the resultant integrals.)



Sec 1



$$s = R\theta$$

$$ds = R d\theta$$

$$dM = ds \cdot \left(\frac{M}{\pi R} \right)$$

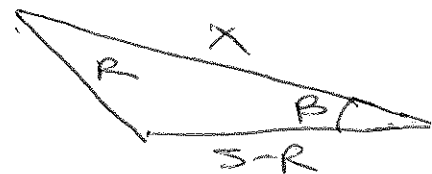
mass density

$$d\vec{F} = \frac{G m dM}{x^2} (-\cos\beta \hat{i} + \sin\beta \hat{j})$$

due to symmetry $\vec{F} = F_0 (-\hat{i})$

$$\Rightarrow F_0 = G m \frac{M}{\pi} \int_{-\pi/2}^{\pi/2} \frac{d\theta}{x^2} \cos\beta$$

$$\cos\beta = \frac{(s-R)^2 + x^2 - R^2}{2x(s-R)}$$



$$\cos(\pi - \theta) = \frac{R^2 + (s-R)^2 - x^2}{2R(s-R)}$$

